notes on 3.1 (interpolation and the Lagrange Polynomial)

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Theorem 3.1 (Weierstrauss Approximation)
Every continuous function on a closed interval can be uniformly approximated by polynomials

Taylor's theorem give polynomial approximations, but they are only accurate around a single point.

Theorem 3.2
Given $x_0, \ldots, x_n$ and a function f, there exists a unique nth degree polynomial $P_n(x)$ that agrees with f on those points. It is given by:

$$P_n(x) = \sum_{k=0}^{n} f(x_i) L_i(x)$$

where

$$L_i(x) = \prod_{j \neq i} \frac{(x - x_j)}{(x_i - x_j)}$$

basically L_i are polynomials with the property that $L_i(x_j) = 0$ if $i \neq j$ and $L_i(x_i) = 1$ if $i = j$

Theorem 3.3
If $x_0, \ldots, x_n$ are distinct in $[a, b]$ and $f \in C^{n+1}[a, b]$, then for each $x \in [a, b]$ there exists $\xi = \xi(x) \in [\min(a_0, \ldots, a_n), \max(a_0, \ldots, a_n)]$ such that

$$f(x) = P(x) + \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

where P is the interpolating polynomial